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HEAT TRANSFER OF A SPHERE IN A CONSTRAINED FLOW OF A VISCOUS FLUID

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Results of an experimental investigation of the average heat transfer of a sphere in the region of Reynolds numbers 0.1–40 are given. The increase in the Nusselt number with increase in the constraint parameter of flow is established. Based on the fractal theory, the influence of the degree of constraint on the heat transfer is physically explained and a quantitative correction is introduced.

Study of the motion of a system of bodies relative to a fluid or gas medium and of the attendant process of heat exchange is of both practical and scientific interest. Diverse examples of the application of results of investigations of this kind are presented in [1] in detail. The first step toward solution of the complex problem is study of the hydrodynamics and heat exchange of bodies of the main geometric shapes: a plate, a cylinder, a sphere, etc. The existing data on theoretical and experimental investigation of the local and average heat transfer of a sphere are analyzed in [2]. The results obtained by M. Sibulkin, H. Murc, B. H. MacAdams, T. Juge, J. Carey, B. D. Katsnel'son, F. A. Timofeeva, L. K. Zhitkevich, L. E. Simchenko, et al. are discussed (references to their publications are given in [2]).

There is an impression that the main questions have been considered and answers have been given to them. However, it should be noted that in certain cases the results of the indicated investigations are contradictory and their authors failed to consider all the factors affecting heat exchange. Practically all of them, including our earlier investigations [3, 4], are characterized by a common drawback: the formulas proposed for calculation contain a great number of empirical constants that limit the region of their application, and the physical meaning of these constants is not revealed.

In the present work, we discuss results of an experimental study of the heat exchange of a sphere in flow of a viscous fluid under constrained conditions. Since experimental data on the heat exchange of bodies for large Reynolds numbers are quite numerous, we focused on an investigation of the structure of the flow for small Reynolds numbers with the aim of elucidating the evolution of instability and the features of the structure of self-organization.

Figure 1 gives the scheme of a vertical water tunnel of the closed type with a closed working portion. Depending on the direction of rotation of the shaft 2, the fluid set in motion by rotation of the screw 1 arrived at plenum chambers 3 or 4 and, passing through honeycombs 5 or 10 and nozzles 6 or 9, followed the contour of a sphere 8 that was installed in the working portion 7. The honeycombs 5 and 10 are plates of 0.020-m-thick organic glass with holes 0.006 m in diameter arranged in staggered order. The nozzles 6 and 9 are calculated from the Vitoshinskii formula with a fivefold contraction.

The working portion 7 of the tunnel is manufactured from transparent organic glass in the form of a rectangular channel of square cross section with dimensions 0.150 × 0.150 × 0.250 m. The transparency of the channel walls enabled us to carry out direct visual observations and to determine the velocity of motion

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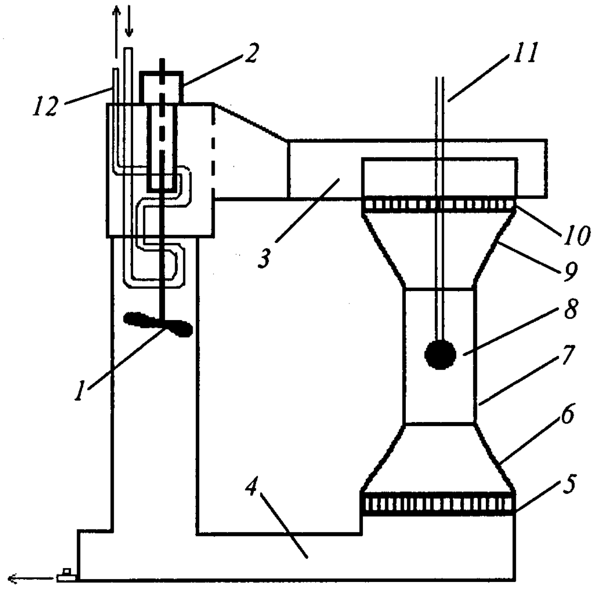

Fig. 1. Schematic diagram of the experimental setup.

of the fluid medium by the method of stroboscopic illumination. The sphere 8 is rigidly fixed; the wires from thermocouples and an electric heater mounted inside the sphere are brought out through a holder 11.

The flow temperature was maintained constant using a coil 12 connected to a TS-24 thermostat.

To study the heat exchange use was made of copper spheres of diameter $h = 0.040$ and 0.025 m with a built-in heating element. The comparatively good heat conduction of copper made it possible to reduce the nonuniformity of the temperature field to 3% for the worst experimental conditions, i.e., for the maximum overheatings of a sphere and velocities of the flow.

In the upper part of the sphere, threaded holes are drilled for a heat-insulating ebonite plug whose surface amounts to less than 1.5% of the sphere surface. The plug has a hole 0.004 m in diameter for fixing the holder and prevents the removal of heat through it.

The heater manufactured in the form of a helix from Nichrome wire is reliably electrically insulated from the sphere.

A 0.0015-m-deep groove is cut on the exterior surface of the sphere along the larger circle: the groove was subsequently filled with epoxy resin. The bottom of the groove is covered with "hot" junctions of copper-constantan thermocouples that are brought out through the holder. The thermocouple junctions are fastened at three points: the leading point, the trailing point, and a point close to the midship section. Special measurements showed that this number of reference points on the sphere surface is quite sufficient to determine the average heat transfer. Cold thermocouple junctions were placed in the flow.

In the stationary regime, with allowance for losses in current-carrying and connecting wires, the average-over-the surface Nusselt number is calculated from the formula

$$\mathrm{Nu} = \frac{IU - I^2 R}{\pi h \lambda_{\mathrm{fl}} \, (T_{\mathrm{b}} - T_{\mathrm{fl}})} \, .$$

The total error in determining the Nusselt number does not exceed 3%.

Glycerin and its aqueous solutions whose viscosity was determined by standard methods was used as the working fluid. In primary processing of experimental data, we calculated the Re number from the maximum velocity on the tube axis. The error in determining the Reynolds number amounts to 1.5–2%.

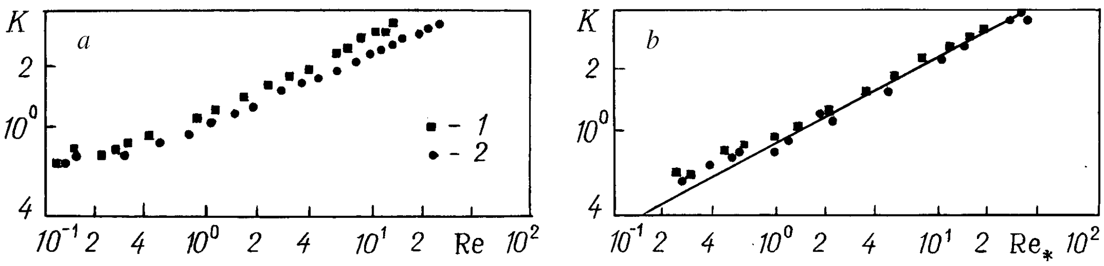

Fig. 2. Influence of the constraint of the flow on the average heat transfer of a sphere: a) primary experimental data; b) allowance for correction (1): 1) χ = 0.235; 2) 0.148. $K \equiv (\mathrm{Nu} - 2)/[\mathrm{Pr}_{\mathrm{fl}}^{1/3}(\mathrm{Pr}_{\mathrm{fl}}/\mathrm{Pr}_{\mathrm{b}})^{1/4}]$.

Figure 2a gives results of measuring the average heat transfer of a sphere for two values of the constraint χ. Since in the case in question our prime interest is with the influence of the constraint factor of the flow, we plotted a dimensionless combination that allows for the thermophysical properties of the medium and the direction of the heat flux on the abscissa axis. The ratios $\mathrm{Pr}_{\mathrm{fl}}/\mathrm{Pr}_{\mathrm{b}} \approx 5.0$ and $\mathrm{Gr} \approx 20$ are maintained constant during the experiments. The plot shows the influence of the constraint factor: the larger the χ, the higher the dimensionless coefficient of heat transfer. In [3, 4], it was proposed that this increase in the heat transfer be taken into account by the introduction of a correction for velocity, i.e., the Reynolds number was calculated from the formula

$$\mathrm{Re}_* = \frac{\mathrm{Re}_{\mathrm{m}}}{1 - 0.95\chi^2}\ .$$

The introduced correction made it possible to satisfactorily generalize most of the data existing at that time. But, first, the number of constants determined from experiment increased still further; second, the correction proposed reflected in practice the influence not only of the constraint itself but also of other factors accompanying the heat exchange.

Advances in fractal theory make it possible to consider in a new manner the influence of the constraint on the hydrodynamics and heat exchange of a body in a channel. The fractal theory of the interaction of a body with a constrained flow of a homogeneous fluid is presented in [5–7]. A calculational formula for the coefficient of resistance of bodies which coincides satisfactorily with experimental data is obtained. The coefficient of resistance of a sphere under constrained conditions is shown to differ from the case of unrestricted flow by the proportionality factor $(1 - \chi^2)^{-D}$. The fractal dimension of the process D is determined in terms of the topological dimension of space and the index of the scaling process of deformation of fluid surfaces by the equation

$$D = d + \gamma - 1\ .$$

We know the values $\gamma = \gamma_0 = 0.7925$ and $\gamma = \gamma_\perp = 0.4650$ that correspond to isotropic and anisotropic scaling processes [8, 9]. The index γ_0 characterizes maximum mixing in three-dimensional flows, while $\gamma_\perp$ characterizes that in flows of the type of a boundary layer with a uniform gradient. The indicated values of γ are the extremum universal characteristics of dynamic chaos since they are calculated for rectangular pulses leading to the resonance stochastization of the medium with a maximum entropy. Whereas the informational entropy – the measure of uncertainty of the rectangular pulses themselves – is minimum, the measure of their certainty – information – is maximum relative to any pulse shape (sinusoidal, delta, etc.) [10]. The confinement of the flow in the gap between solid walls is the determining condition for the anisotropy of its fractalization not only on the body surface but also in the wake of the body. Therefore, when the Reynolds numbers are relatively small we should expect better agreement between theory and experiment for $\gamma = \gamma_\perp$.

Taking into account that the influence of the constraint manifests itself primarily as a change in the velocity distribution, based on the known analogy between the exchange of momentum and heat we propose that a correction for the heat-exchange process be introduced by calculating the effective Reynolds number using the following formula:

$$\mathrm{Re}_* = \frac{\mathrm{Re_m}}{(1-\chi^2)^{D/2}} \,. \tag{1}$$

Then the equation for heat exchange proposed in [11] acquires the form

$$\mathrm{Nu} = C\left[\frac{\mathrm{Re_m}}{(1-\chi^2)^{D/2}}\right]^{\gamma} ,$$

or, taking into account corrections for changes in the thermophysical properties of the medium and comparatively small Reynolds numbers, we obtain

$$\frac{\mathrm{Nu}-2}{\mathrm{Pr_{fl}^{1/3}}\,(\mathrm{Pr_{fl}}/\mathrm{Pr_b})^{1/4}} = C\left[\frac{\mathrm{Re_m}}{(1-\chi^2)^{D/2}}\right]^{\gamma_{\perp}} . \tag{2}$$

The results of processing of the primary experimental data of Fig. 2a using formula (2) are shown in Fig. 2b, where the solid line is the calculated right-hand side of Eq. (2). As is seen from the plot, the coincidence is satisfactory. Unlike the existing empirical formulas, the exponents on the right-hand side have a definite meaning, and the number of constants has been reduced to one, whose value is equal to $C \approx 0.86$ in the region of small Reynolds numbers. A certain excess of the experimental values of the heat-transfer coefficient over the calculated ones is due to the predominant contribution of natural convection to the total heat exchange in this region.

NOTATION

Nu, dimensionless Nusselt number; I, current flowing through the heater, A; U, voltage on the ends of the heater, V; R, total electrical resistance of the current-carrying and connecting wires, Ω; h, diameter of the sphere, m; H, channel width, m; $\chi = h/H$, degree of constraint; λ_{fl}, thermal conductivity of the fluid, W/(m·K); T_{b}, surface temperature of the body, K; T_{fl}, temperature of the medium, K; $\mathrm{Re_m}$, Reynolds number calculated from the maximum velocity; Re_*, effective Reynolds number corrected for the constraint of the flow; $\mathrm{Pr_b}$, Prandtl number calculated for the surface temperature of the body; $\mathrm{Pr_{fl}}$, Prandtl number calculated for the temperature of the medium; Gr, Grashof number; D, fractal dimension; d, topological dimension; γ, scaling index; C, proportionality factor determined from experiment.

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